If $Y_1, Y_2, Y_3, \dots Y_n \sim \text{normal}(\mu, \sigma^2)$

$\bar{Y} \sim \text{normal}(\mu, \frac{\sigma^2}{n})$

- $E[\bar{Y} - \mu] = 0$
- $Var(\bar{Y} - \mu) = Var(\bar{Y}) + Var(-\mu) - 2 * Cov(\bar{Y}, \mu) = \frac{\sigma^2}{n} + 0 + 0$

- $E\left[\frac{\bar{Y} - \mu}{\frac{\sigma}{\sqrt{n}}}\right] = \frac{1}{\frac{\sigma}{\sqrt{n}}} E[\bar{Y} - \mu] = \frac{\sqrt{n}}{\sigma} * 0 = 0$

- $Var\left[\frac{\bar{Y} - \mu}{\frac{\sigma}{\sqrt{n}}}\right] = \frac{1}{\frac{\sigma^2}{n}} Var[\bar{Y} - \mu] = \frac{n}{\sigma^2} * \frac{\sigma^2}{n} = 1$

Law of large number

if $n > 30$, consider $\sigma = s$

$\bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}$

pop. mean

$\lim_{n \to \infty} P(|\bar{Y} - \mu| < \varepsilon) = 1$ for any $\varepsilon > 0$

sample mean

t – distribution (Z dist with unknown variance)

$Z \sim \text{Normal}(0,1)$

$W \sim \chi^2_v$

$t_v \sim \frac{Z}{\sqrt{\frac{W}{n+1}}}$

unknown

$Y \sim n(\mu, \sigma^2)$

s, n given

$P(Y < k)$

$= P\left(\frac{Y - \mu}{\frac{s}{\sqrt{n}}} < \frac{k - \mu}{\frac{s}{\sqrt{n}}}\right)$

$= P\left(t_{n-1} < \frac{k - \mu}{\frac{s}{\sqrt{n}}}\right)$

$\chi^2_v \sim \frac{(n-1)s^2}{\sigma^2}$ where s is the variance of a normal dist

$Z \sim n(0,1)$

$Q = Z^2 \sim \chi^2_1$

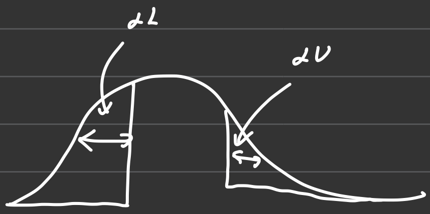
F-distribution

$W_1 \sim \chi^2_{v_1}$ $\quad W_2 \sim \chi^2_{v_2}$

$$F_{(v_1, v_2)} \sim \frac{\frac{W_1}{v_1}}{\frac{W_2}{v_2}}$$

$$\frac{s^2_A}{s^2_B} \sim F_{(n_A-1,\, n_B-1)}$$

Tut Q2, Q3 ignore
Solve 1, 4, 5, 6, 7



$$P(Y < F^{\alpha L}_{(c,d)}) = \alpha$$

$$P(Y > F^{\alpha U}_{(c,d)}) = 1-\alpha$$

$$F^{\alpha U}_{(c,d)} = \frac{1}{F^{\alpha L}_{(c,d)}}$$

Central limit theorem

$$\lim_{n\to\infty} W = \lim_{n\to\infty} \sum_{i=1}^{n} Y_i \sim Normal(n\mu_Y,\, n\sigma^2_Y) \qquad n > 30$$

$$\lim_{n\to\infty} \bar{Y} = \lim_{n\to\infty} \frac{1}{n}\sum Y_i \sim Normal\left(\mu_Y,\, \frac{\sigma^2_Y}{n}\right)$$

Doesn't apply to

$Y = \sum X_i \sim bi(n, p)$

$Var(X_i) = 1^2 * p - E[X_i]^2$
$= p - p^2 = p(1-p)$

$E[Y] = np$
$E[\bar{X}] = E[\frac{1}{n}\sum X_i]$
$= \frac{1}{n}\sum E[X_i] = \frac{1}{n} np = p$

$Var(\bar{X}) = Var(\frac{1}{n}\sum X_i)$

$= \frac{1}{n^2} Var(\sum_i X_i)$
$= \frac{1}{n^2}\left(\sum Var X_i + 2\sum_{1=i<j}^{n} Cov(X_i, X_j)\right)$
$= \frac{1}{n^2}(\sum_i Var X_i) = \frac{1}{n^2} * n * p(1-p) = \frac{p(1-p)}{n}$

Binomial aproximation

$$x \sim binomial(n,p) \approx N(np, np(1-p)) = X_*$$

if $E[X] \geq 5$, $n(1-p) \geq 5$, $0 < p - 3\sqrt{\frac{p(1-p)}{n}}$ and $\pi + 3\sqrt{\frac{p(1-p)}{n}} < 1$

$$P(X \leq x) \approx P(X_* \leq x + \tfrac{1}{n})$$
$$P(X \geq x) \approx P(X_* \geq x - \tfrac{1}{n})$$

Estimators

bias $= B(\hat{\theta}) = E[\hat{\theta}] - \theta$

- want to min. bias

$$
\begin{aligned}
MSE(\hat{\theta}) &= E[(\hat{\theta} - \theta)^2] \\
&= E[\hat{\theta}^2 - 2\hat{\theta}\theta + \theta^2] \\
&= E[\hat{\theta}^2] - 2\theta E[\hat{\theta}] + \theta^2 \\
&= E[\hat{\theta}^2] \underbrace{- E[\hat{\theta}]^2 +}\overbrace{E[\hat{\theta}]^2 - 2\theta E[\hat{\theta}] + \theta^2} \\
&= Var(\hat{\theta}) + B(\hat{\theta})^2
\end{aligned}
$$